1.1

What is Logic?

What do you think of when you hear the word "Logic?"

- Logic is the science of evaluating arguments
- What comes to mind when you think of an "argument"?
- In this class it is a set (group) of statements, one of which is supposed to be supported by the others
- Another way to think about it is to understand that arguments are often used when someone or some group of people is attempting to convince you to believe something.
- Where do you encounter arguments in your life?
- This course is designed to help you to identify which arguments you should accept and which arguments you should reject.
- Law, computer programming, detective work, logic puzzles, standardized tests (SAT, GRE, MCAT, etc.)

What is a statement?

- A claim that is either true or false
- Can anyone give me an example of a statement?
- What types of sentences are not statements?
- Questions
- Commands
- Exclamations
- Proposals
- Suggestions

Examples

- All marmots are mammals. Fido is a marmot, so he must be a mammal.
- Are you going to San Francisco next summer? San Francisco is a larger city than Seattle, but in the summer even more people come to the city. It's about 900 miles from here.
- How many statements?
- Do some statements support others?

Premises & Conclusions

- Premises are the statements that support the others
- The conclusion is the claim being supported by the others
- Conclusion & Premise Indicators (p. 3)
- Indicators are not part of premise or conclusion

Premises & Conclusions

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- Conclusion & Premise Indicators (p. 3)
- Indicators are not part of premise or conclusion

More Examples

- Bob was late today. Thus, because anyone who is late today will be fired, Bob will be fired.
- Mary works on the high seas, since she's a sailor and all sailors work on the high seas.
- George W. Bush should not have been President. He would not condemn torture. He was not skilled enough to lead the country.

Exercises from the text

- 2, 3, 11
- "Therefore" trick
- Note that conclusion can come at the beginning, middle, or end
- There can be any number of premises
- Work on starred problems from 1.1 at home, check your answers
- Look at exercises II, III and IV

1.2

Distinguishing between arguments and non-arguments

There are 3 ways to distinguish between arguments and non-arguments:

- Look for an inferential relationship between some of the statements and one of the statements in a passage.
- Look for the presence of premise or conclusion indicators (note that this does not guarantee that it's an argument)
- Try to identify the passage as one which is clearly recognized as non-argumentative

Typical kinds of non-arguments

- Warnings
  - Look out! He's got a marmot!

More typical kinds of non-arguments

- Pieces of Advice
  - You should get a haircut.
- Statements of Belief
  - Rick Larsen will be reelected in the next election.
- Statements of opinion
  - Today is a beautiful day.

More typical kinds of non-arguments

- Loosely Associated Statements
  - EvCC is the least expensive community college in WA. It has approximately 7,000 different students who attend. It is located 30 miles north of Seattle.
- Reports (even of arguments)
  - The president of the college argued that in order to meet the needs of students in Snohomish county, the size of the Philosophy department would have to be doubled.

More typical kinds of non-arguments

- Expository Passages
  - The community college is an educational institution. It educates a wide variety of people and teaches a wide range of topics.
- Illustrations
  - EvCC offers a wide range of courses. There are classes in philosophy and art, as well as welding and aviation technology.

More typical kinds of non-arguments

- Conditional Statements (A single conditional statement is not an argument)
- If Rick Larsen is defeated in the next election then our congressional district will be represented by a republican.
- Antecedent & Consequent
- If Rick Larsen is defeated in the next election, our congressional district will be represented by a republican.
- Our congressional district will be represented by a republican, if Rick Larsen is defeated in the next election.

Explanations

- Mt. St. Helens erupted because of building geological pressures deep underneath the Earth.
- Mt. St. Helens erupted because of harmonic disalignment.
- Good vs. Bad explanations
- They use indicator words, but aren't arguments
- There are arguments about explanations and explanations of arguments

Distinguishing Explanations from Arguments

- Explanations show why or how something is true
- Arguments attempt to prove that something is true
- I can explain why the sun rose today, I can't argue that it did
- I can argue that the death penalty is morally wrong, I can't explain why it is morally wrong (at least to an audience that isn't all in agreement on this issue).

1.3

Deductive vs. Inductive Arguments

Definitions

- Deductive arguments are those which are written so that the conclusion is meant to follow necessarily from the premises. In other words, if the premises are assumed to be true, the conclusion *is guaranteed* to follow. (At least in the mind of the author of the argument.)
- Inductive arguments are those which are written so that the conclusion is meant to only probably follow from the premises. In other words, if the premises are assumed to be true, the conclusion is *likely* to follow. (At least in the mind of the author of the argument.)

Typical Kinds of Deductive Arguments

- Arguments from Mathematics (but not statistics)
- Al Gore got 3,000,000 votes in Florida in the 2000 election. George W. Bush got 3,000,001 votes in Florida in the 2000 election, therefore Bush got more votes than Gore in Florida in the 2000 election.
- Statistical arguments are often inductive:
  - 90% of all people are right-handed, therefore I'd bet that John Kerry is right-handed.

Typical Kinds of Deductive Arguments

- Arguments from Definition
- This condiment must be chowchow, as you can see, it's a relish of mixed pickles in mustard.

Typical Kinds of Deductive Arguments

- Syllogisms: Two premises and one conclusion
- Categorical Syllogism
  - Each claim begins with "all," "no," or "some"
- Hypothetical Syllogism
  - At least one premise is a conditional statement
- Disjunctive Syllogism
  - One of the premises is a disjunction (either/or)

Which kind of syllogism is it?

- The technology fee will go up anyway, if the students reject the tech fee proposal. The students rejected the tech fee proposal. Hence, the technology fee will go up anyway.
- All computers on this campus are outdated. Thus, all computers on this campus are useless, since all outdated computers are useless.
- I can purchase a Macintosh or a Dell. I won't purchase a Dell, hence I will purchase a Macintosh.

Typical Kinds of Inductive Arguments

- Predictions (these are about the future)
- The Mariners won yesterday and the day before that against the Athletics, so they should beat the A's again today.

Typical Kinds of Inductive Arguments

- Argument from analogy
- These occur when two (or more) things are compared and people conclude that because item (1) has characteristics X, Y, and Z and item (2) also has characteristics X and Y, therefore the second one must have Z as well.

An argument from analogy

- I loved Douglas Coupland's collection of short stories *Life After God*, so his new collection of short stories *Polaroids from the Dead* should be great as well.
- I loved (Z) Douglas Coupland's (Y) collection of short stories (X) *Life After God* (item 1), so his (Y) new collection of short stories (X) *Polaroids from the Dead* (item 2) should be great as well. (I.e., it should have Z as well.)

Typical Kinds of Inductive Arguments

- Inductive Generalizations
- Coming to a conclusion about all the members of a group based on a small sample of that group
- George W. Bush and Dick Cheney have accepted donations from Enron in the past. Thus, it is clear that all Republicans have accepted donations from Enron in the past.
- These can be difficult to distinguish from arguments from analogy. Keep in mind:
  - Analogies conclude about a limited number of things
  - Generalizations conclude about all of a certain group

Typical Kinds of Inductive Arguments

- Arguments from Authority
- Coming to a conclusion based on information from an expert or other authority
- Political commentator David Brooks says that Senator Maria Cantwell has met with leaders of the Palestine Liberation Organization. Thus Cantwell has met with leaders of the P.L.O.

Typical Kinds of Inductive Arguments

- Arguments based on signs
- Similar to arguments from authority, but a sign replaces the person as the source of knowledge
- [Imagine being in a natural history museum.] That placard there says that this piece of rock came from the moon! Wow, pretty amazing to be looking at a piece of the moon, eh?
- It probably is a piece of the moon, but it could also be time to trick the Canadian! So, with no guaranteed conclusion, it's inductive.

Typical Kinds of Inductive Arguments

- Causal Arguments [Not *casual* arguments.]
- Arguments that conclude that one thing caused something else based on evidence
- [Imagine you have natural gas service to your home and when you go home today...] My home smells like rotten eggs, and I have a gas stove, so I must have a natural gas leak.
- Causal Arguments are about specific instances

One kind of argument that can be either Deductive or Inductive

- Arguments from Science
  - Arguments that deal with newly developing science are **inductive**.
  - Arguments that deal with well established science are **deductive**.
  - Another way to think of it is that deductive scientific arguments deal with applications of known scientific laws.

Scientific arguments: Deductive or Inductive?

- [Imagine being in a lab under normal conditions.] Water boils at 100º C, therefore when this water gets to 100º C it will boil. (Deductive)
- [Imagine being in a lab testing a newly discovered substance X, fresh from Mars.] I've been able to boil newly discovered substance X at 75º C twice. Thus, when I bring substance X to 75º C, it will boil. (Inductive)

1.4

Deductive Arguments: Valid/ Invalid, Sound/Unsound

Inductive Arguments: Strong/ Weak, Cogent/Uncogent

Which are "good" and which are "bad" arguments?

1) All mammals have hair. Bobo is a mammal, thus Bobo has hair.
2) Ross Perot was elected President, thus Dan Quayle is Vice President.
3) If the moon is made of green cheese, then Janet Jackson is President. The moon *is* made of green cheese, so Janet Jackson is President.
4) Some politicians are rich and some of the people in this room are rich, thus someone in this room has to be a politician.

Deductive arguments: Valid or Invalid?

- Ask this question first when evaluating deductive arguments:
- If I assume (pretend) that the premise(s) are all true, does the conclusion necessarily have to follow (i.e. is it guaranteed), as the author suggests it does?
- Yes = Valid
- No = Invalid
- Let's go back to the previous arguments

Next: Deductive Arguments: Sound or Unsound?

- If the argument has already been determined to be invalid, it's automatically unsound
- If it's already been determined to be valid, it may be either sound or unsound.
- It will be **sound** if it's valid and all its premises are true, and it will be **unsound** if one or more premises is false.

Three Important Points about Validity

- Valid ≠ True!!!
  - Only **statements**, not arguments are true or false.
  - True premises and a false conclusion always indicates that you have an invalid argument.
  - The Mariners have never been in the World Series and the Mariners have won the highest number of games in a regular season for a team, thus the Mariners are based in Seattle.
  - All true statements, but certainly an invalid argument.

Inductive Arguments: Strong or Weak?

- Ask this question first when evaluating inductive arguments:
- Do the premises, if accepted as true, make the conclusion *likely* to be true, as the author suggests?
- Yes = Strong
- No = Weak
- It's a matter of degree, unlike validity

Going to the Casino

- Let's first of all throw the die
- Now, imagine you're playing Blackjack (21)
- I've got a pair of tens! Therefore I'll probably win if I take one more card.
- I've got a pair of tens! Therefore I'll probably win if I just stay put.
- [You get discouraged and move to roulette]
- Black has come up the last 5 spins! Thus, it'll probably be red on the next one.

*How to win money at a party**

- Since there are at least 23 people here in class today, it is likely that two of us have the same birthday.
- See *Innumeracy* pgs. 26-27 to see the math
- Because there are 535 members of the U.S. Congress, it is likely that two of them were born on the exact same day.
- Because there are 535 members of the U.S. Congress, it is likely that two of them share the same birthday.

***For entertainment purposes only.**

Inductive Arguments: Cogent or Uncogent?

- If the argument is weak, it is automatically uncogent.
- If the argument is strong, it is either cogent or uncogent
  - It is **cogent** if all the premises are true and it's strong
  - It is **uncogent** if at least one premise is false (even if it's strong)
- Go back to the previous arguments

Two more things to remember:

- Strong/Weak and Cogent/Uncogent have nothing to do with whether the conclusion actually turns out to be true or false
- Keep the terms straight, see the chart at the end of 1.4.
- Baker, Rainier and Adams are all volcanoes, so it seems clear that all the mountains in the Cascades are volcanoes.

Exercises

- Determine whether the following arguments are deductive or inductive, what type of deductive or inductive argument and then evaluate: (in)valid, (un)sound or strong/weak, (un)cogent
- 1. The Everett Community College Clipper has an article stating that the college will raise tuition next year by 5%. So, that proves it: tuition is going up next year!
- 2. The college's deficit was $3 million last week but the state just gave the college $1 million that it hadn't planned on receiving, so based just on those numbers, the college deficit is now $2 million.
- 3. Given the way the economy is going, there will be 50% unemployment by January!

3.1

Categorical Logic: Categorical Propositions

Standard Form Categorical Propositions

- All categorical propositions relate two classes (types) of things
- There are four of these:
  - All S are P.
  - No S are P.
  - Some S are P.
  - Some S are not P.
- "S" and "P" are placeholders for the two types of things

There is no "All S are not P"!

- Why? Because it's ambiguous.
- All horses are not animals native to Europe.
- No horses are animals native to Europe?
- Some horses are not animals native to Europe?
- I guarantee you will be tempted to write "All…are not…" at some point-don't do it!

Components of Standard Form Categorical Propositions

- Quantifier
  - "All," "No," "Some"
- Subject Term
  - Between quantifier and "are"
- Copula
  - "are" or "are not"
- Predicate Term
  - Everything after the copula

What "All," "No," and "Some" mean

- "All S are P." means "if there are any S's at all, then they will be P's" (but it doesn't say that S's actually exist).
- "No S are P." means "there are no S's at all that are P's"
- "Some S are/are not P." means "there is at least one S that is/is not a P" (it says that at least one S exists)

3.2

A few other terms to use with Categorical Propositions

Letter Names of the Propositions

- All S are P = "A" statement
- No S are P = "E" statement
- Some S are P = "I" statement
- Some S are not P = "O" statement

Other useful terms for Categorical Propositions

- Quantity
  - Universal/Particular
- Universal: A, E / Particular: I, O
- Quality
  - Affirmative/Negative
- Affirmative: A, I / Negative: E, O

- Identify: quantifier, subject term, copula, predicate term, letter name, quantity, and quality.

1. All fish are things that can breathe in water.

2. No golden retrievers are mammals.

3. Some goldfish bowls are priceless objects.

1.5

Proving arguments invalid with the Counterexample Method

Argument Forms

1) All Collies are dogs. All dogs are mammals, thus all Collies are mammals.
2) If the bus is running, I can get to work. So, I can get to work, because the bus is running.
3) All C are D. All D are M, thus all C are M.
4) If B, then W. So, W, because B.

- Arguments are valid or invalid because of their form

How to extract the argument's form

- For categorical syllogisms:
  - Replace the subject term and the predicate term with a single letter each.
- For other kinds of arguments:
  - Leave: "if," "then," "either," "or," "neither," "nor," "and," "only," and indicator words
  - Other words become single letters (note that in hypothetical syllogisms, single letters stand for entire propositions)

Invalid Arguments

- All Collies are dogs. All Collies are mammals. Thus, all dogs are mammals.
- All C are D. All C are M. Thus, all D are M.
- If tort will be rendered impotent, then corporate power will be unchecked. Corporate power will be unchecked, so it is clear that tort will be rendered impotent.
- If T, then C. C, so T.

Using the Counterexample Method

1) Find the conclusion. (Use indicator words.)
2) Extract the argument's form.
3) Focusing on the conclusion, make substitutions that will cause the conclusion to be false.
4) Plug in what you get into the rest of the problem.
5) Make the remaining substitutions so that the premises are all true.
6) When you have done these five things successfully, you will have shown that the original argument is invalid.

Going back to our examples

- All C are D. All C are M. Thus, all D are M.
- D= mammals M= four legged creatures C= cats
- If T, then C. C, so T.
- T= Elvis Presley was beheaded
- C= Elvis Presley is dead

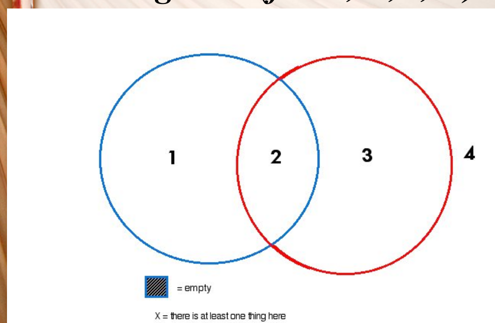
More practice

- All plankton are photovoltaic creatures. Some photovoltaic creatures are reptiles. Thus, no plankton are reptiles.
- All P are V. Some V are R. Thus, no P are R.
- P= cats R= mammals V= felines
- For categorical syllogisms stick with simple substitutions, so it is obvious that your new sentences are true or false.
- For hypothetical syllogisms, stick with dead celebrities.

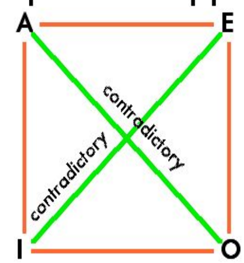
3.3

Venn Diagrams and the Square of Opposition

Venn Diagrams (for A, E, I, O)



The Square of Opposition



"contradictory" means "must have opposite truth values"

Why aren't A and E statements contradictory?

- All cats are mammals.
- No cats are mammals.
- The first is true, the second is false.
- BUT:
- All unicorns are mammals.
- No unicorns are mammals.
- Both are true!

What about I and O statements?

- Some cats are mammals.
- Some cats are not mammals.
- The first is true, the second is false.
- BUT:
- Some cats are tabbies.
- Some cats are not tabbies.
- The first and second are both true.

Immediate Inferences (Args. with one premise)

- Note: the Square does NOT work if the subject and predicate terms are not identical from the premise to the conclusion!
- Only use the Square when the subject and predicate terms are the same in both

Immediate Inferences (Args. with one premise

- It is false that all cows are punctilious creatures, thus it is false that some cows are not punctilious creatures. (Invalid)
- Can' t use the square on the next one!
- All orcas are sea creatures. Thus, it is false that all sea creatures are orcas. (Invalid, based on Venn)

3.4

Conversion, Obversion, and Contraposition

Conversion, Obversion, and Contraposition

- Some unregistered people are not people ineligible for financial aid.
- Huh?
- Some people eligible for financial aid are not registered people.
- We will learn ways to turn sentences like this into ones that make sense.
- But, often we' ll be turning good sentences into stylistically bad ones.

Conversion

- Switch the subject and predicate terms
- That's it!
- Some doctors are not talented individuals.
- Some talented individuals are not doctors.
- No frogs are mammals.
- No mammals are frogs.
- A, O are not logically equivalent when converted
- E, I are logically equivalent when converted

Term Complements

- The term complement is everything something is not (given the context)
- Chairs?
- Cars?
- Inefficient people?
- Gaseous things?
- People who are not teachers

Obversion

- Change the quality (but not the quantity) and replace the predicate with its term complement
- All hot dogs are cholesterol containing foods.
- No hot dogs are cholesterol free foods.
- A, E, I, O are all logically equivalent when obverted

Contraposition

- Switch the subject and predicate terms and replace each with their term complements
- No fiends are garbage haulers.
- No non-garbage haulers are non-fiends.
- Some patient people are not successful people.
- Some unsuccessful people are not impatient people.
- A, O are logically equivalent when contraposed
- E, I are not logically equivalent when contraposed

The Ultimate Challenge

- All lizards are reptiles.
- Contrapose this
- All non-reptiles are non-lizards.
- You can obvert any categorical proposition to make it easier to diagram.
- No non-reptiles are lizards.

4.3 Review: Valid or Invalid?

- It is false that no marmots are reptiles. Thus, it is false that some marmots are not reptiles.
- Some songs are not enjoyable sounds. So, it is false that all enjoyable sounds are songs.

3.6

Translating nonstandard categorical propositions into standard form

Why worry about it?

- Most categorical propositions do <u>not</u> occur in standard form, so we have to translate them in order to work with them in the way we have been in previous sections.
- There are ten main ways in which a categorical proposition may differ from standard form, and we'll look at additional examples of each (Hurley has many other good examples.)

- Terms without nouns
  - Some lights are bright.
  - Some lights are bright things.
- Nonstandard Verbs
  - All thieves will rob you.
  - All thieves are persons who will rob you.
- Singular Propositions (one of the categories is a specific person, place, thing, or time)
  - Everett is beautiful.
  - All places identical to Everett are beautiful places.
  - All people identical to…, All things identical to…, All times identical to…

Watch out for these words

- Any time you see one of the following words in the middle of a sentence, before translating put the word and anything that follows it at the beginning of the sentence and then translate:
- When, where, what, whenever, wherever, whatever, if, only, *the* only, and unless
- Also "Rule for A propositions" lists on pg 183

Adverbs and pronouns

Jay Inslee whispers when he talks.

When Jay Inslee talks, Jay Inslee whispers.

All times Jay Inslee talks are times Jay Inslee whispers.

Note that I didn't use "…identical to…"

Why?

Unexpressed Quantifiers

- A good first step in translation is to ask yourself which quantifier you want to use
  - Sport utility vehicles are expensive.
  - All suv's are expensive items.
  - Suv's are sold by this auto dealer.
  - Some suv's are vehicles sold by this auto dealer.

Nonstandard Quantifiers

- Few, *a* few, not every, anyone, All…are not…
- A few pens are green.
- Some pens are green things.
- Few pens are green.
- Some pens are green things and some pens are not green things.
- Not every person present is eligible.
- Some people present are not people who are eligible.
- Anyone who is a fan of the Mariners will be disappointed.
- All people who are fans of the Mariners are people who will be disappointed.

Conditional Statements (if…then, unless)

- You try this one:
  - A tiger is not happy if it bites.
  - All tigers that bite are unhappy animals.
  - No tigers that bite are happy animals.
  - All tigers that bite are not happy animals?!
- "Unless" means "if…not"
  - Unless tigers are fed, they will bite.
  - If a tiger is not fed, then it will bite.
  - All tigers that are not fed are tigers that will bite.

- Only:
- Make sure "only" is at the beginning and use "all" and switch the order of the terms. (There's a problem with the translation hints box.)
- Only men are playing in the NFL.
- All men are people playing in the NFL?
- All people playing in the NFL are men.
- *The* Only:
- Again, make sure "the only" is at the beginning and use "all," but <u>don't</u> switch the order of the terms.
- Birds are the only large animals that can fly.
- The only large animals that can fly are birds.
- All birds are large animals that can fly?
- All large animals that can fly are birds.

Exceptive Propositions: All except S are P & All but S are P

- All cats except tigers are friendly animals.
  - No tigers are friendly animals and all cats that are nontigers are friendly animals.
- Now you try one:
- All employees but teachers can receive bonuses.
  - No teachers are employees who can receive bonuses and all employees other than teachers are employees who can receive bonuses.

No…except

- No colleges except EvCC are community colleges located in Everett.
- It means "The only community college located in Everett is EvCC."
- All community colleges located in Everett are colleges identical to EvCC.

How to translate

- First look for one of the key words in the tan box at the end of the section
- Make sure the required words are at the beginning of the sentence before translating.
- If there are no key words in the sentence, ask yourself which quantifier should be used to translate the sentence while preserving the meaning.

Translation Checklist

- Make sure that you have one of our three quantifiers beginning your sentence.
- Make sure that you have one of the two copulas in the sentence.
- Make sure that you could convert the sentence and have it still make sense. (noun in predicate)
- Make sure that you have eliminated all key words in the tan box at the end of this section.
- Don't use "identical to" when not needed.
- Make sure that your new sentence means the same thing as the old one!
- Watch out for "All…are not!"

Exercises

- Do #2 from Exercise set I
- Do non-starred problems in Exercise Set II

5.1

Propositional Logic: Logical Operators

Differences between Categorical and Propositional Logic

- Capital letters no longer stand for types of things, but rather complete propositions
- However, a capital letter stands for a "simple" proposition, which does not contain any of the logical operators we will be discussing
- ALWAYS use capital letters, not lowercase (we'll need those in chapter 7)

Five new friends

- "~" Tilde (used for "not")
- "•" Dot (used for "and")
- "v" Wedge (used for "or")
- "⊃" Horseshoe (used for "if…then")
- ≡ Triple Bar (used for "if and only if")
- "☺" Just Kidding

Tilde (Negations)

- Hot dogs aren't healthy foods.
- ~H
- What does "H" stand for?
- It is not the case that bananas are vegetables.
- ~B
- What does "B" stand for?

Dot (Conjunctions)

- Salami is tasty and apples are fruity.
- **S • A**
- It is false that Salami is tasty but sausage is tasty.
- **~S • G**
- Both John and Bob are not employed.
- **~J • ~B**
- Compare with **~(J • B)**

Wedge (Disjunction)

- John or Bob is employed.
- **J v B**
- Neither [Not either] John nor Bob is employed.
- **~(J v B)**
- John or Bob is employed, but not both.
- **(J v B) • ~(J • B)**
- Either John or Bob is not employed.
- **~J v ~B**

Horseshoe (Conditionals)

- If today is Tuesday, then my assignment is due.
- **T ⊃ D**
- **My assignment is due, if today is Tuesday.**
- This assignment is late only if the instructor changed the due date.
- **L ⊃ C**
- See the complete list at the end of 5.1

Necessary and Sufficient Conditions

- Go back to 1.2, exercise set VI to practice
- You'll see this on the exam.
- SUN or **S** ⊃ **N** is how we can know how to translate sufficient and necessary conditions
- Use examples from the exercises above

The Triple Bar (Biconditionals)

- Tom is a bachelor if and only if he is unmarried and over 18.
- **T ≡ (~M · O)**
- Tom's being a bachelor is a sufficient and necessary condition for taking this poll.
- **T ≡ P**
- What do we do with parentheses?

John they bachelor is

- "John they bachelor is"
- It's not well formed, right?
- Check out engrish.com for more examples

Well formed formulas

- The same thing can happen with statements in propositional logic.
- Look at 5.1 Exercise Set III
- Which ones are well-formed formulas (wffs)?

Can you find the main operator?

- In math, I think we can fairly easily determine the main operator of a mathematical equation
- What would you say is the main operator of: 2 • (5+3)? (It' s the last operator you' d work on if you were working on the problem.)
- It' s the same with propositional logic.
- A ⊃ (~B ≡ C)
- ~[A v (C ⊃ G)]
- You can only have one main operator
- See 5.2 Exercise Set I

5.2

The truth tables for the propositional operators

The Truth Tables (see front cover of text)

~	p		p	•	q		p	v	q		p	⊃	q		p	≡	q
F	T		T	T	T		T	T	T		T	T	T		T	T	T
T	F		T	F	F		T	T	F		T	F	F		T	F	F
			F	F	T		F	T	T		F	T	T		F	F	T
			F	F	F		F	F	F		F	T	F		F	T	F

5.3, 5.4, 5.5

Truth Tables and what they' re good for

How to set up a truth table

- First of all, you want to list all of the *different* letters to the left
- Then you need to know how many rows you'll need.
- The formula is 2^n = the number of rows you'll need, with n being the number of different letters in the statement
- (A v B) ⊃ A

Definitions for single propositions

- Once you complete your truth table for a single statement you should look at the main operator column to determine if it's:
- Contingent (sometimes T and sometimes F)
- A Tautology (all T)
- A Self-Contradiction (all F)
- (A ≡ B) ⊃ A

Definitions for multiple statements

- What you do is compare the main operator columns to determine if two or more statements are:

1. Logically equivalent (same exact values)
2. Contradictory (opposite truth values)
3. Consistent (both are true in at least one row)
4. Inconsistent (none of the above)

More practice

Now try a truth table for three statements:

1. ~(A v B)
2. A ⊃ ~A
3. C

Using Truth Tables to Make Sense of the World

- Word problems
- Try 5.3 Exercise set II: 3

Exercises Translate and determine T/F

- McCain won the Presidency or he is not a Republican. (M, R)
- Washington was assassinated if France didn't bomb Pearl Harbor. (W, F)
- Obama won the Presidency and Biden won the Vice-Presidency. (O, B)

Be sure to circle main operator value!

Using truth tables to determine if an argument is valid/invalid

- Remember what I told you back in chapter 1: any deductive argument that allows for the possibility of all true premises and a false conclusion is necessarily INVALID.
- So, if the earlier example were instead an argument, with #3 being its conclusion, we could tell that it was invalid if it had a row in it where under the main operators you had all true premises and a false conclusion.

Now you try a couple

- Premise 1: $\sim D \vee C$
- Premise 2: $\sim C \vee E$
- Conclusion: $C \equiv D$
- Premise 1: $A \supset \sim B$
- Premise 2: $\sim B \bullet A$
- Conclusion: $\sim A \vee (B \bullet A)$
- Review Exercise set IV from 6.2

How about this one?

- Premise 1: [(B ⊃ A) • E] ≡ F
- Premise 2: A • ~G
- Conclusion: F v H
- Ouch!
- 6 Different letters = how many rows?
- A lot!
- (64 to be more precise)

Indirect Truth Tables

- We don't want to have to do huge truth tables when there's an easier way.
- See your handouts for the indirect truth table method.
- Not only does it work with determining validity, it can also be used to show if sets of statements are consistent, by attempting to make all the statements true, rather than the last one false.

Indirect Truth Tables

- Make the conclusion false
- Plug values in to premises
- Make premises true under main operator
- Start with easiest premise
- Plug in values as you get them
- Continue until you get all true premises or a contradiction
- If you can't get all true premises and there's only one way to make the conclusion false it's valid, if you can get all true premises, it's invalid

Using Indirect Truth Tables to test for consistency

- The only difference here is that we are trying to see if all of the claims can be true at the same time
- So, you start by making a claim true (hopefully a claim that can only be made true in one way)
- Continue until you can make all the claims true at the same time, or you get a contradiction

6.1

The Rules of Inference

The Rules of Inference

- Modus Ponens (MP)
- If the Mariners have won today's game then they're in the World Series. The Mariners have won today's game, thus they're in the World Series.
- Modus Tollens (MT)
- If the Mariners have won today's game then they're in the World Series. The Mariners are not in the World Series, thus they didn't win today's game.

More rules of inference

- Pure Hypothetical Syllogism (HS)
- If the students pass their exams, then they will pass the class. If they pass the class, then they will graduate. Hence, if the students pass their exams, then they will graduate.
- Disjunctive Syllogism (DS)
- Either Spain will win today's game or they will lose it. The Spaniards will not win, hence the Spaniards will lose today's game.

Some comments about the rules of inference

- All of these rules go one-way only
- They're all valid argument forms
- The p, q, r, and s all stand for wffs, any wffs at all, including the same wffs.
- These rules ONLY apply when the main operator of the rule is the same as the main operator of the proposition you're applying the rule to.

Try this one

1. ~X v ~Y
2. ~X ⊃ Z
3. ~Z
4. Y v W // W

6.2

The Rules of Inference Continued

Constructive Dilemma (CD)

- If Elizabeth Warren wins the next Presidential election, then we'll have a Democratic President, and if Marco Rubio wins the next Presidential election, then we'll have a Republican President. Either Warren or Rubio will win the next Presidential election, hence we'll either have a Democratic or a Republican President.

Simplification and Conjunction

- Simp
- Obama is President and Biden is Vice President. Thus, Obama is President.
- Conj
- Obama is President. Biden is Vice President. Thus, Obama is President and Biden is Vice President.

Addition

- Add
- Obama is President.
- Hence, Obama is President or Clinton is President.
- Or: Hence, Obama is President or Thog is President.
- Or: Hence, Obama is President or the moon is made of green cheese.

6.3

The Replacement Rules

The Replacement Rules

- These are a bit different from the rules of inference
- They work on parts of lines
- They work both ways
- They are logical equivalences
- A ⊃ (B ⊃ C) / B // C?
- No!
- But, (A • B) v C / ~(B • A) // C works with Com to make it: ~(A • B)

Demorgan's Rule (DM): with tildes and wedges or tildes and dots, you'll use Demorgan's lots and lots

- Note how these mean the same thing…
- Not both Warren and Rubio will win the next Presidential election.
- Either Warren won't win or Rubio won't win the next Presidential election.
- Neither Barack Obama nor George Bush will win the next Presidential election.
- Obama won't win the next Presidential election and Bush won't win the next Presidential election.

Commutativity (Com)

- Note how these mean the same thing
- Com:
- Warren or Rubio will win.
- Rubio or Warren will win.
- Bush and McCain are Republicans.
- McCain and Bush are Republicans.

Associativity and Distribution

- Assoc: Just note that when you' re dealing with all wedges or all dots, the parentheses move around (consider the lunch in my bag)
- Dist: Note how the dots and wedges change places and the outside term gets distributed to each part. (consider going from Lynnwood to Seattle)

Double negation

- DN:
- It' s not the case that Everett is not located in Snohomish county.
- Everett is located in Snohomish county.
- Distribution is not Demorgan' s and vice versa.

6.4

The Replacement Rules Continued

Transposition

- Trans:
- If Warren wins, we’ll have a Democratic President.
- If we don’t have a Democratic President, then Warren didn’t win.

Material Implication and Material Equivalence

- Imp:
- If Warren wins, we’ll have a Democratic President.
- Either Warren won’t win, or we’ll have a Democratic President.
- Equiv:
- It’s the only rule that deals with the triple bar, see my discussion of the triple bar truth table to see how this rule works.

Exportation

- Exp:
- If you’re driving like a maniac and a state trooper sees you, then you’ll get a ticket.
- If you’re driving like a maniac, then if a state trooper sees you, then you’ll get a ticket.

Tautology

- Taut:
- We already had half of this rule
- Everett is a city.
- Everett is a city and Everett is a city.
- You can have the black car or you can have the black car.
- You can have the black car.

6.5 & 6.6

Conditional and Indirect Proof

Try a problem using only the 18 rules

- X ⊃ Y // X ⊃ (X • Y)
- Can't do it, can you? But it's clearly valid!
- So we'll use conditional proof to derive the conclusion
- You can use conditional proof to get a conditional statement whenever you need one.
- Assume the antecedent of the conditional statement you're trying to get, then focus only on getting the consequent of the conditional you're trying to get.

Indirect Proof

- Deriving a contradiction is a sign that one of your premises must be false, because contradictions are always false.
- See 7.2 Exercise set III: 10 to see this.
- So, in indirect proof we're trying to get a contradiction, i.e. something, dot, the same thing with a tilde in front of it.

Which are contradictions?

(A • B) • ~(A • B)

(A v B) • (~A v B)

A • ~~A

~A • A

A • ~B

~~~A • A
~~~

Do this one using CP or IP

1. (A v B) ⊃ D
2. (D v ~ G) ⊃ C   / A ⊃ C

This problem requires a minimum of 8 steps.

7.1

Translating ordinary language into the symbols of Predicate Logic

Predicate Logic

- It's a combination of categorical and propositional logic
- Predicates are uppercase letters A-Z and stand for types of things
- Individual Constants are lowercase letters a-w (NOT x, y, or z) and stand for unique persons, places, things, or times
- Variables are lowercase x, y, and z and range over individual things

Universal Statements

- Universal statements (A, E statements)
- We need the Universal Quantifier to express these: (x) or (y) or (z)
- It means "for all x" or "for any x"
- It is used 99% of the time with a ⊃
- So, translate "All rabbits are mammals."
- (x)(Rx ⊃ Mx)
- How about "No rabbits are fish?"
- (x)(Rx ⊃ ~Fx)

Particular Statements

- Particular statements (I, O)
- We need the Existential Quantifier to express these: (∃x) or (∃y) or (∃z)
- It means "There exists an x such that…"
- It is used 98% of the time with a •
- Translate "Some Senators are Republican."
- (∃x)(Sx • Rx)
- Now try "Some Senators are not Republican."
- (∃x)(Sx • ~Rx)

What's wrong with these?

- (x)(Rx • Mx)
- Universal quantifier shouldn't go with a dot

- (∃x)(Rx ⊃ Mx)
- Existential quantifier shouldn't go with a horseshoe

Free and Bound Variables

- Free variables are variables that do not have a quantifier which applies to them.
- If a variable is free it is ambiguous (we could ask: all x? or some x?)
- Quantifiers apply just like tildes did in propositional logic

    (x)Ax ⊃ (x)Bx (all bound)

    (∃x)(Ax • Bx) v Cx (Cx has a free variable)

    Cy ⊃ By (both "y"s are free)

Translation Checklist

1. Check for any individual constants. (If you have them, you may not need a quantifier.)
2. Determine which quantifier to use.
3. If you're using (x), use a "⊃", for (∃x) use a "•", if anything.
4. Make sure there are no free variables in your translation.
5. Check to see that the translation has the same meaning as the original sentence.

Try these:

- Fish are mammals. (F, M)
  (x)(Fx ⊃ Mx)
- Some fish are endangered species. (F, E, S)
  (∃x)[Fx • (Ex • Sx)]
- Poisonous fish is not served at this restaurant. (P,F,S)
  (x)[(Px • Fx) ⊃ ~Sx]
- A fugu is a poisonous fish. (F, P, I)
  (x)[Fx ⊃ (Px • Ix)]

Check the translation hints

- See the translation hints in 3.6 & 5.1
- Try "Politicians and lawyers are interested in the election." (P, L, I)
- (x)[(Px v Lx) ⊃ Ix]

Don't use a dot on this one, because then you have to be a politician and a lawyer at the same time to be interested in the election.

7.2

Dealing with Quantifiers

Universal Instantiation

- UI: Basically there are no restrictions on using this rule
- **(x)Fx→Fy or Fa**
- "(x)Fx" is any statement in predicate logic with a universal quantifier as its main operator
- "y" can be any variable, even "x"
- "a" can be any individual constant
- When the universal quantifier is the main operator, you can use this rule to pull it off and change the variable to any letter a-z

Universal Generalization

- UG: Can only use this rule under special circumstances
- **Fy→(x)Fx but NOT Fa→(x)Fx**
- Consider "Aaron is a teacher.": Ta
- It would be invalid to conclude (x)Tx
- So, you can only add on the universal quantifier when you have variables, and it must end up being the main operator of the final statement

Existential Instantiation

- EI: There are severe restrictions on using this rule
- **(∃x)Fx→Fa but NOT Fy and the individual constant must be new**
- If you did (∃x)Fx→Fy, then you could use UG and get (x)Fx, which is invalid [We could go from "Teachers exist." to "Everything is a teacher."]
- Take "Marlee Matlin is deaf." [1. Dm] and "There is a deaf Senator." [2. (∃x)(Dx•Sx)]
- Can we use EI to get 3. Dm•Sm?
- That would be invalid!

Existential Generalization

- EG: There are no significant restrictions on using this rule
- **Fa→(∃x)Fx and Fy→(∃x)Fx**
- This rule allows you to put on the Existential Quantifier as long as you make the quantifier the main operator and you are changing all of the same letters
- E.g. you can't go from "Marlee Matlin is deaf and Maria Cantwell is a Senator" [Dm•Sc] to "There is a deaf Senator." [(∃x)(Dx•Sx)]

Which uses of the rules work?

1. (x)(Fx ⊃ Gx)
2. Fy ⊃ Gx UI 1

1. (x)(Fx ⊃ Gx)
2. Fx ⊃ Gx UI 1

1. (∃x)Fx ⊃ (∃x)Gx
2. Fa ⊃ Ga EI 1

1. (∃x)(Fx•Gx)
2. Fa•Ga EI 1

Which uses of the rules work?

1. Ba
2. (∃x)Cx
3. Ca EI 2

1. (x)(Bx•Dx)
2. By•Dy UI 1
3. (∃x)(Bx•Dx) EG 2

1. Ab•Bb
2. (∃x)Ax•(∃x)Bx EG 1

1. Fa
2. (x)Fx UG 1

1. (x)(Bx ⊃ Gx)
2. (∃x)(Bx•Dx)
3. Ba ⊃ Ga UI 1
4. Ba•Da EI 2

7.3

The Quantifier Negation Rules

The QN Rules

Everything is Brahman. **(x)Bx :: ~(∃x)~Bx**

It's not true that there exists something which isn't Brahman.

Not everything is Brahman. **~(x)Bx :: (∃x)~Bx**

There exists something which isn't Brahman.

The QN Rules

- Martians do exist. **(∃x)Mx :: ~(x)~Mx**
- It's not the case that you can take anything and it won't be a Martian.

Martians don't exist. **~(∃x)Mx :: (x)~Mx**

Take any x, it won't be a Martian.

7.4

Conditional and Indirect Proof in Predicate Logic

One new restriction on UG

- Look at this problem:
- **1. (∃x)Ax / (x) Ax**
- **We can derive it using IP, but it's clearly invalid! (assume ~Ax)**
- **To avoid this, we must not use UG on a free variable in our assumption while indented.**

Exercises

- Let's try these:

1. (∃x)Bx ⊃ (∃x)Gx
2. (∃x)Gx v (∃x)Bx /(∃x)Gx

1. (x)[(Ax v Bx) ⊃ (Cx • Dx)] /(x)(Ax ⊃ Cx)

7.5

How to show arguments in Predicate Logic are Invalid

The Counterexample Method

- Review your notes from 1.5
- Remember that if we can show that an argument allows for all true premises and a false conclusion, it is necessarily invalid.
- So, in Predicate Logic we will make substitutions for the predicates and any individual constants to get all true premises and a false conclusion

Exercises

1. (x)(Ax ⊃ Bx) / (∃x)Bx • (∃x)Ax

All Elbonians are things from Elbonia.

C: Things from Elbonia exist and Elbonians exist.

1. (x)[(Px v Hx) ⊃ Dx]
2. Pm /(x)(Dx ⊃ ~Px)

1. If something is a human or a whale then it is a mammal.
2. Mike is a human.

C: No mammals are humans.

The Finite Universe Method

- Are these arguments invalid even if Kevin Costner dies?
- Even if all the cars on the planet disappear?
- Two step process:
  - Translate into a one-member universe
  - Do indirect truth tables to get all true premises and a false conclusion (see 6.5 and your handout on this)
  - If that doesn't work with one member, add another member and repeat until you can.

Exercises

1. (x)(Ax ⊃ Bx) / (∃x)Bx • (∃x)Ax (already did this one)
2. Now you try this one:

1. (x)(Ax ⊃ Bx)
2. (∃x)(Ax • Cx) / (x)Bx